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ATMOSPHERE

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R. S. ROCKWOOD

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF
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THE ULTRAVIOLET TRANSMISSION COEFFICIENT OF THE EARTH'S ATMOSPHERE

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ABSTRACT

Thermocouple measurements have been made at four different altitudes in the neighborhood of Albuquerque, New Mexico, of the intensity of solar radiation transmitted by a silver filter. These have been used to determine values of the transmission coefficient defined by $I = I_0 a^m$ where I and I_0 are respectively the observed and incident energies, a the transmission coefficient and m the air mass traversed in units of the vertical air path directly overhead at sea level and normal pressure. The results, corrected by the use of Hann's empirical equation for the effect of water in the path, gave for a at 3140 m, 0.47; at 2059 m, 0.47; at 1769 m, 0.46; and at 1556 m, 0.43. The last result, from observations on the New Mexico campus is low probably because of dust scattering; the others are in agreement within the errors. The theoretical value calculated from Rayleigh's law of molecular scattering, and using a new value ($\lambda 3240$) for the wave length of maximum solar energy transmitted by a silver film, is 0.479. The conclusion is, that at least above 1700 m altitude, and for clean dry air, the Rayleigh law for pure molecular scattering is adequate to account for the atmospheric depletion of solar radiation in the silver transmission band. The somewhat discordant results of earlier investigators are discussed.

I. INTRODUCTION

The solar radiation which reaches the surface of the earth is weakened in intensity by its passage through the atmosphere. A study of the nature of the various causes of depletion and of their relative significance

has commanded much interest and a large amount of data has been obtained by various workers. For the visible part of the solar spectrum the previous data seem satisfactory; in the ultraviolet, however, the various observations are highly discordant. The aim of the present work has been to obtain additional data in the ultraviolet with the object of throwing further light upon the variations in the existing measurements.

The chief causes of depletion of solar radiation in the atmosphere are: (1) absorption by water vapor; (2) absorption by gases of the atmosphere; (3) scattering by dust and haze; (4) scattering by water vapor; (5) scattering by air molecules.

The absorption by water vapor and by gases is of interest chiefly in the ultraviolet, where ozone and oxygen cause absorption and in the red where the great water vapor bands set in. In the region from $\lambda 3000$ to $\lambda 5000$ absorption is a negligible factor in the weakening of the sun's radiation.

The scattering by water vapor and dust is nearly independent of wave length as may be seen from the white color of clouds and haze. The amount of water vapor in the air may be measured and its influence corrected for. The effect of haze is a residual factor after others have been removed. In the present work where interest was centered largely on the molecular scattering, haze scattering was avoided by observing only on the clearest days.

The law of molecular scattering was given by Lord Rayleigh¹ from consideration of an elastic solid theory, and later on the basis of the electromagnetic theory of light. According to this scattering formula the intensity of light after passage through a gas is given by

$$I = I_0 e^{-kx} \quad (1)$$

where I is observed intensity of light of wave length λ ; I_0 , intensity incident on the medium; X , thickness of the medium; $k = 32\pi^3(\mu - 1)^2 / 3N\lambda^4$; μ , index of refraction of the gas; N , number of gas molecules per unit volume, λ , wave length of incident light.

Many tests have shown that this formula satisfactorily explains the scattering of light by gases in general and by the atmosphere for visible light. Fowle² at the Mt. Wilson Observatory, has made extensive observations on the intensity of the solar radiation at all wave lengths from $\lambda 3700$ to $\lambda 17,000$. The amount of water vapor was carefully measured and its effect eliminated. The results showed conclusively that in

¹ Rayleigh, *Phil. Mag.*, 47, 375; 1899.

² Fowle, *Astrophys. J.*, 38, 392; 1913.

the region from $\lambda 3700$ to $\lambda 5000$, where there were no absorption effects, that the Rayleigh scattering formula was adequate to account completely for the depletion of the solar radiation by clean dry air. In the region of shorter wave lengths, however, the experimental data are not in entire agreement. The classical investigations of Fabry and Buisson³ have shown that a strong absorption, which sets in about $\lambda 3100$, increases rapidly with decreasing wave length and limits the sun's spectrum at about $\lambda 2900$, may be explained as due to a layer of about 3 mm of ozone in the earth's atmosphere—probably at high altitudes since very little ozone is observed near the earth's surface.⁴ Between $\lambda 3100$ and $\lambda 3700$ the question as to whether or not Rayleigh scattering is adequate to account for the depletion of solar radiation by pure dry air has received varying and quite discordant answers. Dawson, Granath, and Hulbert⁵ by a photographic method measured the absorption of the atmosphere for a 400 m path near the ground and found the Rayleigh formula sufficient to account for all weakening of intensity for wave lengths longer than $\lambda 2900$. On the other hand Schaeffer⁶ who also measured the atmospheric absorption by a photographic method for horizontal air paths near the earth, ranging in length up to 8000 m, found that for wave lengths shorter than $\lambda 3400$ the Rayleigh scattering alone called for a greater transmission than observed. The chief study of the depletion of solar radiation in this region is that of Pettit⁷ who on the basis partly of his own observation and partly on the analysis of those of others concluded that for the light near $\lambda 3200$, transmitted by a silver filter, the Rayleigh scattering calls for less transmission than actually observed at altitudes above 1800 m, and for greater at lower altitudes. None of these ultraviolet researches took account of the effects of water vapor or dust in the atmosphere which would decrease the expected transmission and thus might account for results in which the observed transmission was less than calculated.

To attempt a further study of the role of Rayleigh scattering in atmospheric transmission the present study was undertaken in the neighborhood of the University of New Mexico at Albuquerque. Using a method similar to Pettit's but correcting for water vapor scattering the coefficient of transmission of the atmosphere for that part of the

³ Fabry and Buisson, *Astrophys. J.*, 54, 297; 1921.

⁴ Cötz and Ladenburg, *Naturwiss.* 18, 373; 1931.

⁵ Dawson, Granath and Hulbert, *Phys. Rev.* 34, 136; 1929.

⁶ Schaeffer, *Proc. Amer. Acad. of Arts and Sci.* 57, 385; 1921.

⁷ Pettit, *Nat. Research Council Bull.*, No. 61, 101; 1927.

solar spectrum transmitted by a silver film has been determined from measurements made at altitudes of 1556 m, 1769 m, 2059 m, and 3140 m.

Climatic conditions in the southwestern United States are unusually favorable for such a study. High altitude, low humidity, cloudless skies and freedom from the dust of large cities make the atmosphere remarkably clear. In the fall and winter months there are few high winds which might raise dust in the air and so cause loss of radiation by dust scattering. At Albuquerque, New Mexico and in the nearby mountains are found altitudes accessible in an automobile that range from 1556 m (5100 feet) on the campus of the University of New Mexico to 3140 m (10,300 feet).

Albuquerque is situated on the east bank of the Rio Grande at an altitude of about 5000 feet. From the University campus east of the city the mesa slopes gradually and uniformly eastward for a distance of about 8 miles to the base of the Sandia Mountains where the altitude is 6000 feet. The west side of the mountains or the side facing Albuquerque is precipitous, the altitude changing in a horizontal distance of 2 miles to 10,300 feet. The eastern slope is not so steep and here a forest service road leads to the crest of the range. The stations at which observations were made are located *A* at the crest of the range, altitude 10,300 feet; *B* on the west side of the mountain, altitude 6750 feet; *C* on the open mesa, altitude 5800 feet; and *D* on the campus of the University of New Mexico, altitude 5100 feet. Between these stations there is little difference in the character of the country which is semi-arid to arid. The rainfall in Albuquerque varies from 4 to 10 inches per year, a large part of which falls in the months of July and August. In the mountains there is a slightly larger amount of precipitation but in the fall almost no rain falls and the prevailing winds which blow from the desert keep the humidity low.

II. EXPERIMENTAL DETAILS

A diagram of the equipment is shown in Fig. 1. It consisted of a thermopile, *D* located at one end of a brass cylinder of such length that a quartz lens, *B*, at the other end would focus incident radiation on one junction. In the path of the ray between the lens and thermopile was placed a quartz disk, *C*, on one side of which a thin silver film had been deposited by sputtering. The brass cylinder carrying the lens and thermopile was placed inside a Dewar flask, *E*, to reduce errors arising from changes in temperature of the air. This element was inclosed in a

hard rubber cylinder, *F*, as a protection against breakage. Over a window in one end of the protective casing was placed a "Corex A" glass filter, *A*, which also had a thin silver film on its inner side, to insure against unfiltered radiation through possible pin holes in either silver film. The "Corex A" filter was used as an additional filter to keep all longer wave-length radiation out of the equipment. The casing was provided with a finder to insure the proper focusing of the radiation on the junction of the thermopile and the whole unit was given a polar mounting on the base of a heliostat. A Leeds and Northrup high sensitivity galvanometer measured the thermoelectric currents.

To avoid any depletion due to haze, observations were taken only on days when the sky was free from clouds and when the mountains to the north, west, and south, whose distances were from 65 to 100 miles,

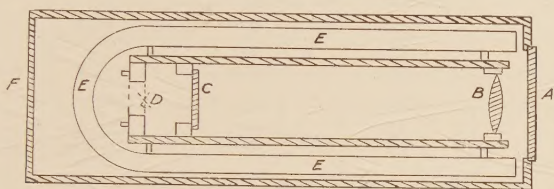


FIG. 1. Cross section of receiver: *A*, silvered Corex filter; *B*, quartz lens; *C*, silvered quartz plate; *D*, thermocouple; *E*, Dewar flask; *F*, hard rubber case.

gave a sharp outline against the sky. Observations were begun by 10:00 A.M. if possible and were continued at half hour intervals until 3:30 P.M. For each observation from three to six readings were made, the number of readings taken depending on the agreement of the results. Focusing of the sun's image on the thermopile was accomplished by slowly shifting the receiving set so that the image of the sun moved across the junction of the thermopile. The maximum deflection of the galvanometer was taken as the reading. The zero reading of the galvanometer was observed before and after each exposure and the mean taken for the zero of the reading. Since small changes in the leveling would result in a serious difference of deflection the galvanometer was leveled with a small spirit level and the leveling checked several times each day. Difference in temperature in the galvanometer parts were guarded against by a covering of corrugated card board which completely covered the galvanometer except for a small window left to enable deflections to be observed. At intervals of about one hour readings were taken of the temperatures on the wet and dry bulb thermometers.

A total of 119 observations on 19 days were made at the station on the

campus of the University of New Mexico, elevation 1556 m; 15 on two days at Sunset View Gas Station, elevation 1769 m; 81 on 12 days at Carlito Springs, elevation 2059 m; 54 on 5 days at Kiwanis Cabin, elevation 3140 m.

Table 1 is a typical set of observations and computed results that was made at elevation 1556 m, October 17, 1931.

TABLE 1. *Observations and reduced data for October 17, 1931. Elevation 1556 m, University of New Mexico Campus.*

Hour	Zenith distance ϕ	Air mass (B/B_0)sec ϕ	Galvanometer deflection d	Relative intensity $I=dc$	Log I
10:30	47.85°	1.25	6.78	140.3	2.1483
11:00	45.5°	1.19	6.80	140.7	2.1483
11:30	44.2°	1.17	6.98	144.8	2.1608
12:00	43.85°	1.17	6.26	130.0	2.1139
1:30	49.5°	1.29	5.00	103.0	2.0128
2:00	53.35°	1.40	4.66	96.0	1.9823
2:30	57.6°	1.56	4.09	83.5	1.9217
3:00	62.4°	1.81	3.08	63.0	1.7993

Barometer reading, 63.6 cm. Vapor pressure, $e=0.557$ cm. c , a constant.

ANALYSIS OF DATA

For comparison with the results of others it is somewhat more convenient to calculate from the data not the absorption coefficient, k , of Eq. (1) but the transmission coefficient of the atmosphere defined by

$$I = I_0 a^m \quad (2)$$

where " a " is the transmission coefficient and m is the air mass traversed by the light in terms of unit air mass. The unit air mass is the thickness of the atmosphere directly overhead at sea level and at normal atmospheric pressure. This definition differs from one often used in which a unit air mass is defined for each altitude as the air thickness directly overhead at normal pressure. According to our use of the term, for elevations above sea level, the fractional part of the unit mass directly overhead would be given by the ratio B/B_0 , of the observed barometric pressure to standard sea-level pressure. Since outside the tropics observations on the sun directly overhead can never be made, the mass of the air traversed by the transmitted radiation is greater than the mass directly overhead by an amount which can be found by multiplying the overhead mass by the secant of the sun's zenith distance, ϕ .

This calculation rests on the assumptions that the portion of the earth's surface is flat and that the layers of air are horizontal, which for a body the size of the earth leads to no large errors for angles up to 75° . The air mass for any barometric pressure and zenith distance is thus

$$m = (B/B_0) \sec \phi. \quad (3)$$

For the calculation of the air mass both the barometric pressure and the sun's zenith distance are required. A small aneroid barometer whose readings were checked with a weather bureau standard mercurial barometer was used to get the barometric pressure. Use of the following equation was made to determine the zenith distance of the sun.

$$\sin h = \sin B \sin \theta + \cos B \cos \theta \cos \alpha$$

h , altitude of the sun; B , latitude of the station; θ , declination of the sun; α , hour angle of the sun. The zenith distance, ϕ , is equal to $90^\circ - h$.

In this formula only B , the latitude of the station, is known directly but both θ and α are obtainable with the help of a nautical almanac.

Eq. (2) may be written in logarithmic form

$$\log I = \log I_0 + m \log a. \quad (4)$$

If $\log I$ be plotted against m the points should fall on a straight line of slope $\log a$ which extrapolated to zero air mass would give the value of I_0 , the incident energy. " a " is not quite a constant but for clean air is really the product of two coefficients,

$$a = a_a \cdot a_w^q. \quad (5)$$

Where a_a is the transmission coefficient for dry air and a_w is that for water vapour with exponent q —the thickness in centimeters of the layer of water which would be formed by the precipitation of the water vapor in unit air path.

a_w is for our observations small and study of the data has shown that for purposes of determining I_0 it is sufficiently accurate to regard a as constant. The straight lines so determined by plots of Eq. (4) all lead to the same values of I_0 within the errors of observation. Fig. 2 shows these lines for observations at altitudes 1556 m and 2059 m. The value of I_0 so fixed is of course in arbitrary units of the galvanometer deflections, that is, it is the galvanometer deflection which would be obtained with our apparatus if it could be placed outside the atmosphere.

Using this value of I_0 a value may be calculated for " a " for each observation. To obtain a_a (Eq. (5)) it is necessary to correct for the scat-

tering by water vapor and hence to know the amount of water in the air path. To determine the amount of water present, we have used the empirical equation devised by Hann⁸ to fit observations made, at many localities and elevations, of the rate of decrease of vapor pressure with altitude. Naturally it holds only for average conditions. We believe, however, that the steady atmospheric conditions at Albuquerque in the fall with prevailing winds from the desert justify the application of the formula. The vapor pressures, however, were at all times so low

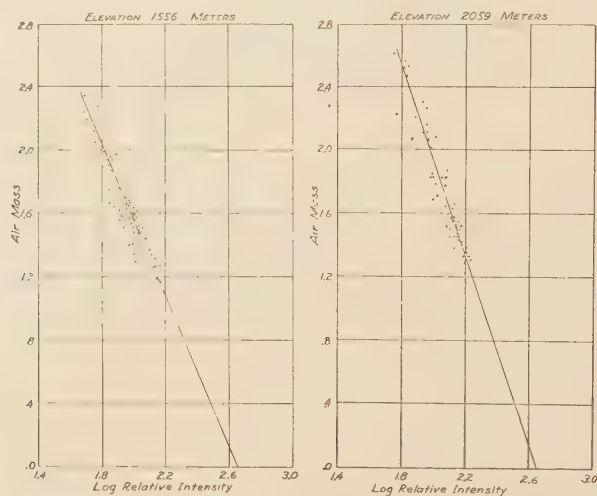


FIG. 2. Intensity-air mass curves.

that the correction for water is small and a large uncertainty here would affect the final result only slightly.

Hann's equation is

$$Q_1 = 2.3e_0(1 - 10^{-h/5000}) \quad (6)$$

where d_1 is the quantity of water expressed in cm of precipitable water between sea level and altitude h given in meters, and e_0 is the sea-level vapor pressure, in centimeters. For an altitude h equal to the height of the limits of the atmosphere the equation reduces to

$$Q_2 = 2.3e_0. \quad (7)$$

If we substitute for h the altitude of the University of New Mexico and subtract (6) and (7) we will get an expression for the quantity of water above station D to the limit of the atmosphere.

⁸ Hann, *Lehrbuch der Meteorologie*, p. 230; 1915.

$$Q = Q_2 - Q_1 = 2.3e_0 - 2.3e_0(1 - 10^{-1556/5000})$$

$$Q = 1.17e_0.$$

From Hann's relation between sea-level vapor pressure, e_0 and the vapor pressure at an altitude h , e_w , we get

$$e_w = e_0 10^{-1556/6500}$$

$$e_0 = 1.73e_w.$$

Substituting this value for e_0 in the equation for q we have

$$Q = 1.93e_w.$$

This gives the quantity of water in the vertical column of air above station D in terms of the observed vapor pressure. Thus at noon October 17, 1932 the vapor pressure was observed as 0.557 cm and Q was 1.08 cm.

In a similar way Q was determined for the other stations.

Station A —elevation 3140 meters— $Q = 1.64 e_w$

Station B —elevation 2059 meters— $Q = 1.85 e_w$

Station C —elevation 1769 meters— $Q = 1.87 e_w$.

Formula (5) is written for unit air mass and the Q which is the amount of water above the observation point must be multiplied by B_0/B to give the q needed for formula (5). On the days on which observations were taken the vapor pressures at station D ranged between 0.13 to 0.61 cm and the corresponding values of Q from 0.5 to 1.4 cm.

Eq. (5) may be written in logarithmic form as $\log a = \log a_a + q \log a_w$. If now $\log a$ be plotted against q , the points should fall on a straight line which may be extrapolated to zero water to give the value of $\log a_a$. Fig. 3 shows such straight lines for the observations at 1556 and 2059 m. The values of the absorption coefficient so obtained are:*

at 3140 m $a = 0.47$

at 1769 m $a = 0.46$

at 2059 m $a = 0.47$

at 1556 m $a = 0.43$.

For the three higher elevations there is very good agreement in the values of " a ". The coefficient observed at 1556 m shows the influence of dust and smoke of the nearby city which has caused about 10 percent decrease in transmission.

The scattering of the points about the line in Figs. 2 and 3 represents an uncertainty of about 5 percent. Partly this is due to inherent difficulties in outdoor observations under atmospheric conditions which cannot

* In the following pages of this article a will be written for a_a .

be controlled; in part the scattering is due to errors in the use of Hann's formula to calculate the amount of water in the path. These uncertainties are certainly not more than is usual and unavoidable in such observations and do not leave the final value of " a " in doubt by more than 4 or 5 percent.

The values of " a " at the higher elevations, which have been corrected for water vapour and which are only slightly, if at all, influenced by haze and dust, should be directly comparable with e^{-KX} of Rayleigh's formula. The only constant in doubt in $K = 32\pi^3(\mu - 1)^2/3N\lambda^4$ is λ .

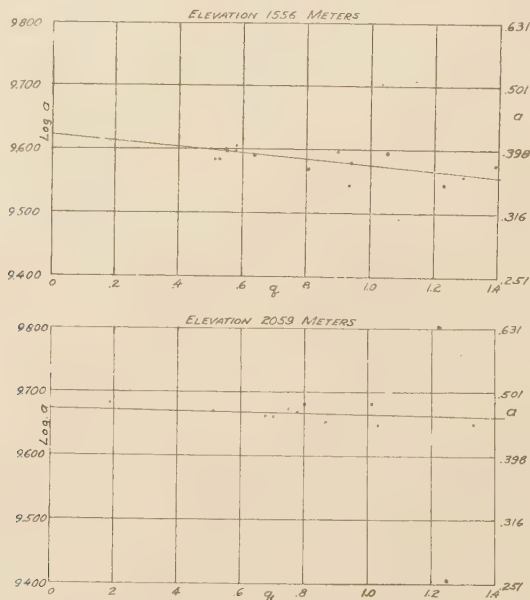


FIG. 3. Transmissibility of radiation through moist air.

Pettit⁷ in his work with the solar radiation transmitted by a silver film used $\lambda 3200$ in computing K . It is true that $\lambda 3200$ is the center of the transmission band of silver.⁹ However, the solar energy curve falls off very rapidly in this region and the wave length of maximum energy transmitted by the silver filter must be greater than $\lambda 3200$. The position of this maximum may be determined by plotting for each wave length the product, C , of the transmission of silver for that wave length by the relative energy of the solar spectrum, as is done in Fig. 4. The solar energy curve, A , is taken from measurements by Brian O'Brien, published by Forsythe and Christison¹ and the silver transmission, B , from

⁹ Hagen and Rubens. Ann. d. Physik, 8, p. 432; 1902.

¹⁰ Forsythe and Christison, G. E. Rev., 32, p. 662, 1929.

Hagen and Rubens.⁹ It is seen that the wave length of maximum energy transmission is about $\lambda 3240$. We have accordingly used that value in computing K . For this wave length $(\mu - 1) = 2.88 \times 10^{-4}$,¹¹ N is taken as $2.707 \times 10^{19,12}$ and X as 7.99×10^5 .¹³ With these values $e^{-KX} = a = 0.479$.

In Table 2 are collected for comparison our values of "a" at the different elevations together with those reported by Pettit, which have been corrected to unit air mass. It will be noted that Pettit's value for "a" at Mt. Whitney is in almost perfect agreement with the calculated value.

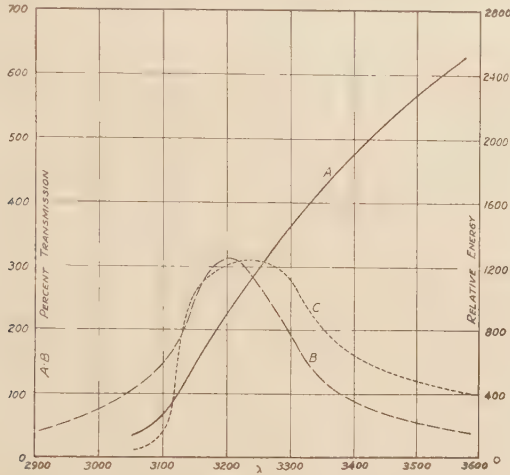


FIG. 4. Maximum solar energy transmitted by silver near 3200. A, solar energy curve; B, transmission curve for silver; C, transmitted energy.

TABLE 2. Results in comparison. Rayleigh scattering by unit air mass.

$a = e^{-kz}$	$\mu - 1 = 2.88 \times 10^{-4}$
$k = 32\pi^3(\mu - 1)^2/3N\lambda^4$	$N = 2.707 \times 10^{19}$
	$\lambda = 3.24 \times 10^{-8}$
	$\chi = 7.99 \times 10^5$
$a = 0.479$ "a" for unit air mass	

Observed at Albuquerque		Given by Pettit	
elevation	"a"	elevation	"a"
3140 meters	0.47	Mt. Whitney 3500 meters	0.475
2059 meters	0.47	Mt. Wilson 1740 meters	0.405
1769 meters	0.46	Tucson 760 meters	0.338
1556 meters	0.43	Pasadena 258 meters	0.320
		Oxford, Eng. 64 meters	0.312

¹¹ Smithsonian Physical Tables, 7th edition, p. 293; (1923).

¹² Birge, Phys. Rev. Sup. 1, p. 1; 1929.

¹³ Kimball, Monthly Weather Review 55, p. 155; 1927.

Pettit himself used in calculation of " a ", $\lambda 3200$, as well as different values of the other constants, and obtained for a , 0.448. As pointed out above it seems to us that the effective wave length should be $\lambda 3240$. Pettit's coefficients at elevations lower than Mt. Whitney are all smaller than would be expected from the formula. This may be accounted for by the fact that there is no evidence that there was any correction applied for the water vapor in the atmosphere. For the Mt. Whitney observations that fact would have little effect, since above an altitude of 3500 m there is very little water in the atmosphere.

In the work done at Albuquerque the agreement between the observed coefficients at altitudes 3140 m, 2059 m and 1769 m and the calculated molecular scattering coefficient is well within the error of observation. The low observed value at 1556 m altitude is undoubtedly due to effect of dust and smoke from the city. It is our intention to extend this work to lower elevations, and with improved methods of determining the amount of water vapor in the air path at low altitudes.

Our results show that at least for altitudes above 1700 m, and for clean dry air, the Rayleigh law for pure molecular scattering is adequate to account for the atmospheric depletion of solar radiation in the transmission band of silver. It is also our belief that this is the case at all altitudes when the observations are corrected for the effects of scattering by dust and water vapor.



